I Functions, Graphs, Limits

* One- and Two-Sided Limits
  - Using Algebra
  - Intuitively
* Asymptotic & Unbounded Behavior
  - Asymptote = $\lim_{x \to +\infty \text{ or } x \to -\infty} f(x)$
  - Relative magnitude of functions & rate of change
* Continuity
  - understand/define using limits
  - IVT
  - EVT

II Derivatives

* Concept: graphically, numerically, analytically
  - instantaneous rate of chg
  - limit of difference quotient "slope formula"
  - Differentiability $\Rightarrow$ Continuity

II (continued)

* Derivative at a point
  - slope of a curve at a point. slope of a tangent
  - equation of a tangent line at a point (a,b)

    $y-b=m(x-a)$

    also "local linear approximation at a point" [[use tangent line to approx. function value nearby]]

    $y=b+m(x-a)$
  - approx instantaneous rate of change with AVERAGE rate of chg
* Derivative as a function
  - characteristics of graphs of f' and f''
  - $f'>0 \Rightarrow f$ increasing ; $f'<0 \Rightarrow f$ decreasing
  - MVT and geometric interpretation
  - verbal descriptions translated to eqns involving derivatives (and vice-versa)
* Second Derivatives
  - characteristic of f, f', f'' graphs
  - $f''>0 \Rightarrow$ concave up ; $f''<0 \Rightarrow$ concave down
  - points of inflection. Pts where concavity changes

II (continued)

* Applications of Derivatives
  - Analysis of curves: inc/dec; monotonicity; concavity;
    - = critical points: f' is zero/undefined; possible extremum
    - = possible pt of infl: f'' " ; check for pt of inf.

Yo! ONCE you can do this for function, you RULE

- relative vs. absolute extrema
- optimization
- related rates & other models of RATE OF CHANGE
- implicit differentiation: especially to find deriv. of inverse $f(f^{-1})(x) = x$
- interpretation of derivative. EG. position, velocity, acceleration, SPEED
- geometric interpretation of deriv [slope fields] sign of v, sign of a
- differential equation

distance- always positive

ANSWER to a differential equation is a CONTINUOUS FUNCTION

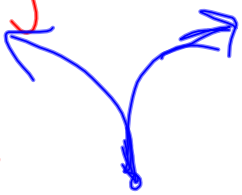
displacement- pos/neg

* Computation of derivatives
  - product - quotient - chain - b^x - $\log_b x$
  - trig - inverse trig

* Pictures of function:

$|x|$, x, x^2, x^3, $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{2}{3}}$

e^x, $\ln x$, trig, inverse trig



III Integrals

* interpretation and properties
  - define as limit of Riemann sums
  - accumulation $\int_a^b f'(x)\,dx = f(b) - f(a)$
    (total change is integral of instantaneous rate of change)

* Applications
  - position, velocity, acceleration
  - distance $= \int_a^b |v(t)|\,dt$; displacement $= \int_a^b v(t)\,dt$
  - areas
  - volumes with known cross-section
    - find area of cross-section; integrate
  - average value of a function
  - extend to other situations
    - create Riemann-like sum
    - limit $\Rightarrow$ definite integral

$\Rightarrow$

* FTC

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \quad ; \quad \int_a^b f'(x)\,dx = f(b) - f(a)$$

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III Integrals (cont)

* Techniques of anti-differentiation
  - memory
  - substitution of variables
* Applications of antidifferentiation
  - w/ initial condition
* Numerical approximation
  - right Riemann sum
  - left Riemann sum
  - midpt Riemann sum
  - trapezoidal sum

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